TRANSACTIONS

OF THE

ROYAL IRISH ACADEMY.

I.—*On the Determination of the Intensity of the Earth's Magnetic Force in absolute Measure. By the* Rev. Humphrey Lloyd, D. D., *Fellow of Trinity College, Dublin, and Professor of Natural Philosophy in the University; F. R. S.; V. P. R. I. A.; Honorary Member of the American Philosophical Society, and of the Batavian Society of Experimental Philosophy.*

Read January 9, 1843.

THE attention of mathematicians and experimentalists has been, for some time past, directed to the means of determining the intensity of the earth's magnetic force in absolute measure. These means consist, it is well known, in observing the time of vibration of a freely-suspended horizontal magnet, under the influence of the earth alone, and then employing the same magnet to act upon another, which is also freely-suspended, and noting the effects of its action combined with that of the earth. From the former of these observations we deduce the *product* of the horizontal component of the earth's magnetic force into the moment of free magnetism of the first magnet,—from the latter, the *ratio* of the same quantities; and, the product and the ratio being thus known, the two factors are absolutely determined. The former part of this process involving no difficulty which may not be overcome by due care in observing, we shall confine our attention, in the present communication, to the latter.

Two methods have been proposed for this second observation, one by Poisson, and the other by Gauss. The method of Poisson consisted in observing the *time of vibration* of the second magnet, under the combined action of the first and of the earth, the acting magnet having its axis in the magnetic meridian passing through the centre of the suspended magnet. In the method of Gauss, which is now universally adopted, we observe the *position of equilibrium* of the second magnet, resulting from the action of the same forces. The acting magnet being placed transversely with respect to the suspended one, the latter is deflected from the meridian, and the amount of this deflection serves to determine the ratio of the deflecting force to the earth's force. The position chosen by Gauss for the deflecting magnet is that in which its axis is in the right line passing through the centre of the suspended magnet, and perpendicular to the magnetic meridian, in which case the tangent of the angle of deflection is equal to the ratio of the two forces. From this ratio it remains to deduce that of the magnetic moment of the deflecting bar to the earth's force.

The difficulty of this process arises from the form of the expression of the force of the deflecting bar. This force being expressed by a series descending according to the negative odd powers of the distance, with unknown coefficients, it is evident that observation must furnish as many equations of condition, corresponding to different distances, as there are terms of sensible magnitude in the series; and from these equations the unknown quantities are to be deduced by elimination. Now, the greater the number of unknown quantities thus eliminated, the greater will be the influence of the errors of observation on the final result; and if, on the other hand, the distance between the magnets be taken so great, that all the terms of the series after the first may be insensible, the angle of deflection becomes very small, and the errors in its observed value bear a large proportion to the whole.

It fortunately happens, that at moderate distances (distances not less than four times the length of the magnets) all the terms beyond the second may be neglected. The expression for the tangent of the angle of deflection is thus reduced to two terms, one of which contains the inverse cube of the distance, and the other the inverse fifth power; that is, if u denote the angle of deflection, and D the distance,

$$\tan u = \frac{Q}{D^3} + \frac{Q}{D^5};$$

in which Q and Q′ are unknown coefficients, the former of which is double of the ratio sought. Accordingly, the method recommended by Gauss consists in observing the angles of deflection, u and u', at two different distances, D and D′, and inferring the coefficient Q by elimination between the two resulting equations of condition.

The object of the present paper is to point out the means by which the quantity sought may be obtained, without elimination, from the results of observation at *one* distance only; and thus not only the labour of observation be diminished, but (which is of more importance) the accuracy of the result increased. Before entering on this, however, it will be expedient to ascertain the amount of the probable error in the received method.

The coefficient of the first term, obtained by elimination between the two equations of condition above alluded to, is

$$\mathrm{Q} = \frac{\mathrm{D}'^5 \tan u' - \mathrm{D}^5 \tan u}{\mathrm{D}'^2 - \mathrm{D}^2}.$$

The distances being greater than four times the length of the magnets, the angles of deflection are small, and there is, approximately, $\tan u = u \tan 1'$, $\tan u' = u' \tan 1'$, u and u' being expressed in minutes; and making $\mathrm{D}' = q\mathrm{D}$, the preceding expression becomes

$$\mathrm{Q} = \mathrm{D}^3 \tan 1' \frac{q^5 u' - u}{q^2 - 1}.$$

The probable errors of u and u' are equal; and, by a well known theorem of the calculus of probabilities, the probable error of Q is

$$\Delta\mathrm{Q} = \mathrm{D}^3 \tan 1' \frac{\sqrt{q^{10}+1}}{q^2-1} \Delta u.$$

In determining the ratio of this error to the quantity itself, we may observe that there is, approximately, $q^3 u' = u$, and

$$\mathrm{Q} = \mathrm{D}^3 \tan 1' . u;$$

and dividing the formula last found by this,

$$\frac{\Delta\mathrm{Q}}{\mathrm{Q}} = \frac{\sqrt{q^{10}+1}}{q^2-1} \frac{\Delta u}{u}.$$

It appears from the preceding theorems, that the value of $\Delta\mathrm{Q}$, corresponding to a given value of Δu, varies with the assumed ratio of the distances, q; and

that, in order to apply the method most advantageously, this ratio must be taken in such a manner, that the probable error, ΔQ, shall be the smallest possible. This condition gives

$$\frac{d}{dq}\left(\frac{\sqrt{q^{10}+1}}{q^2-1}\right)=0;$$

whence we obtain the following equation for the determination of q:

$$3q^{10}-5q^8-2=0.$$

In order to solve this equation we may observe that, q being greater than unity, the last term of the equation may, in a first approximation, be neglected in comparison with the others; so that we have, approximately,

$$3q^2-5=0, \qquad q=\sqrt{\frac{5}{3}}=1\cdot 3.$$

And setting out from this value, we find, by any of the known methods of approximation,

$$q=1\cdot 32;$$

or $1\frac{1}{3}$, very nearly. Accordingly the smaller distance, r, being determined by the condition that the third term of the series shall be insensible, the greater distance should be $1\cdot 32\,r$.

If we substitute this value of q, in the expression for $\frac{\Delta Q}{Q}$ above obtained, we find

$$\frac{\Delta Q}{Q}=5\cdot 563\,\frac{\Delta u}{u};$$

from which we can calculate the least probable error corresponding to any given angle of deflection, the probable error of reading being known.*

* In the Dublin Magnetical Observatory the deflecting bar hitherto employed is 12 inches in length, and the least deflecting distance therefore 4 feet. The deflection produced by it at this distance is about 3° 56′; and the probable error of observation does not exceed 5″. Hence, in this case,

$$\frac{\Delta Q}{Q}=\frac{5\cdot 563}{236\times 12}=\cdot 0020.$$

The absolute intensity, X, varies inversely as the square root of Q; so that

$$\frac{\Delta X}{X}=\frac{1}{2}\frac{\Delta Q}{Q}.$$

Consequently, the resulting probable error in the determination of the absolute intensity, made according to the usual method, is, at this Observatory, about the $\frac{1}{1000}$th part of the entire quantity.

Now let us suppose the term containing the fifth power of the distance to vanish, in the expression for the deflecting force. The value of Q will then be reduced to

$$Q = D^3 \tan u;$$

in which we may take $\tan u = u \tan 1'$, as before. Hence there is

$$\frac{\Delta Q}{Q} = \frac{\Delta u}{u};$$

and the probable error is less than in the usual method in the ratio of 1 to 5·563, even when the latter is employed in the manner most conducive to accuracy. Accordingly, if by any means the coefficient of the inverse fifth power of the distance can be annihilated, or rendered so small that the term shall have no sensible influence, the accuracy of the results will be increased more than five-fold, and, at the same time, the observations being taken at one distance only, the labour of observation will be halved.

The same advantages will be gained, if, the coefficient of the inverse fifth power of the distance retaining a sensible value, the ratio of the two coefficients may be known *à priori*. Let

$$Q' = hQ,$$

h being a known quantity. In this case the expression for the tangent of the angle of deflection becomes

$$\tan u = \frac{Q}{D^3}\left(1 + \frac{h}{D^2}\right);$$

and the coefficient sought is obtained, from the result of observation at a single distance, by the formula

$$Q = \frac{D^3 \tan u}{1 + hD^{-2}}.$$

It is evident that the probable error of Q thus obtained, arising from an error in the observed deflection, is the same as in the case last considered, and therefore between 5 and 6 times less than in the ordinary method.

The object of the following investigation is to point out the means of attaining these advantages.

Let the axis of the deflecting magnet be supposed to lie in the right line joining the centres of the two magnets, and let the axis of the suspended magnet make the angle ψ with that line. Then, if X and Y denote the forces exerted

by the deflecting magnet, upon any element of free magnetism, m, of the suspended magnet, in the direction of the line connecting it with the centre of the deflecting magnet, and in the line perpendicular to it, respectively, their moment to turn the magnet round its point of suspension will be

$$r\{\mathrm{X}\sin(\phi+\psi)+\mathrm{Y}\cos(\phi+\psi)\};$$

r denoting the distance of the particle m of the suspended magnet from its centre, and ϕ the angle which the line connecting this particle and the centre of the deflecting magnet makes with the axis of the latter. Now, I have elsewhere shown* that, if we include the terms involving the fifth power of the distance, the values of X and Y are

$$\mathrm{X}=\frac{2m}{a^3}\cos\phi\left\{\mathrm{M}+\frac{\mathrm{M}_3}{a^2}(5\cos^2\phi-3)\right\},$$

$$\mathrm{Y}=\frac{m}{a^3}\sin\phi\left\{\mathrm{M}+\tfrac{3}{2}\frac{\mathrm{M}_3}{a^2}(5\cos^2\phi-1)\right\};$$

a being the distance of the particle m of the suspended magnet from the centre of the deflecting magnet, and M and M_3 denoting, respectively, the integrals corresponding to $\int mrdr$, $\int mr^3dr$, for the deflecting magnet, taken between the limits $r=\pm\frac{1}{2}$ length. Now

$$\sin\phi=\frac{r}{a}\sin\psi;$$

so that, extending the approximation to the term involving the fifth power of the distance only, we must make $\sin\phi=0$, $\cos\phi=1$, in the coefficient of that term. The preceding expressions are thus reduced to

$$\mathrm{X}=\frac{2m}{a^3}\left(\mathrm{M}\cos\phi+\frac{2\mathrm{M}_3}{a^2}\right),\qquad \mathrm{Y}=\frac{m}{a^3}\mathrm{M}\sin\phi;$$

and, substituting, the moment of these forces to turn the magnet is

$$\frac{mr}{a^3}\left\{\mathrm{M}\left(3\sin\phi\cos\phi\cos\psi+(2-3\sin^2\phi)\sin\psi\right)+\frac{4\mathrm{M}_3}{a^2}\sin\psi\right\};$$

or, eliminating ϕ by the relation between it and ψ,

$$\frac{mr}{a^3}\sin\psi\left\{\mathrm{M}\left(2+3\frac{r}{a}\cos\psi-3\frac{r^2}{a^2}\sin^2\psi\right)+\frac{4\mathrm{M}_3}{a^2}\right\}.$$

* Transactions of the Royal Irish Academy, vol. xix. p. 163.

Now, if D denote the distance between the centres of the two magnets,

$$a^2 = \text{D}^2 + r^2 - 2\,\text{D}r\cos\psi.$$

Wherefore, developing the inverse powers of a in series ascending according to the powers of $\frac{r}{\text{D}}$, stopping at the inverse fifth power of the distance, and substituting in the expression for the moment above given, it becomes

$$\frac{mr}{\text{D}^3}\sin\psi\left\{\text{M}\left(2+9\cos\psi\,\frac{r}{\text{D}}+6\,(5\cos^2\psi-1)\,\frac{r^2}{\text{D}^2}\right)+\frac{4\,\text{M}_3}{\text{D}^2}\right\}.$$

This being the moment of the force exerted by the deflecting magnet upon a single particle, m, of the suspended magnet, the moment of the force exerted upon the entire magnet is obtained by multiplying by dr, and integrating between the limits $r = \pm\, l$, l being half the length of the suspended bar.* The magnetism being supposed to be distributed symmetrically on either side of the centre of the suspended magnet, and the axis of suspension to pass through that centre, we have

$$\int_{-l}^{+l} mr^2 dr = 0.$$

Accordingly, denoting the integrals $\int_{-l}^{+l} mrdr$, $\int_{-l}^{+l} mr^3 dr$, by M' and M_3', the expression for the moment of the whole force is

$$\frac{2\,\text{MM}'}{\text{D}^3}\sin\psi\left\{1+\left(2\frac{\text{M}_3}{\text{M}}+3\,(5\cos^2\psi-1)\,\frac{\text{M}_3'}{\text{M}'}\right)\frac{1}{\text{D}^2}\right\}.$$

When there is equilibrium, this moment must be equal to that of the force exerted by the earth upon the suspended magnet. Let X be now taken to denote the horizontal component of the earth's magnetic force. The moment of the force exerted by that component upon the particle m of the suspended magnet is

$$m\text{X}r\sin u\,;$$

u denoting the deviation of the axis of the magnet from the direction of the force. Multiplying by dr, therefore, and integrating, the total moment is

* We have here assumed, that the effect of a magnet is the same, with respect to any point at a moderate distance, as if the magnetic elements in each section perpendicular to its axis were all concentred in the axis; or, in other words, that the integration with respect to the other dimensions of the bar introduces no new term into the integral. There is no difficulty in proving that such is the case, the magnetic elements being supposed to be distributed symmetrically with respect to the axis.

$$\text{X}\text{M}' \sin u.$$

Hence the equation of equilibrium is

$$\text{X} \sin u = \frac{2\text{M}}{\text{D}^3} \sin \psi \left\{ 1 + \left(2 \frac{\text{M}_3}{\text{M}} + 3 \left(5 \cos^2 \psi - 1 \right) \frac{\text{M}_3'}{\text{M}'} \right) \frac{1}{\text{D}^2} \right\}.$$

There are two cases of this solution which demand our consideration.

In the method of GAUSS, the deflecting magnet is perpendicular to the magnetic meridian, and therefore $\psi = 90° - u$. In this case, then, the preceding equation becomes

$$\text{X} \tan u = \frac{2\text{M}}{\text{D}^3} \left\{ 1 + \left(2 \frac{\text{M}_3}{\text{M}} - 3 \frac{\text{M}_3'}{\text{M}'} + 15 \sin^2 u \frac{\text{M}_3'}{\text{M}'} \right) \frac{1}{\text{D}^2} \right\}.$$

Accordingly, the term containing the fifth power of the distance is composed of two parts, one of which is constant, while the other varies with the angle of deflection ; so that, if there were no means of determining *à priori* the values of the ratios, $\frac{\text{M}_3}{\text{M}}$, $\frac{\text{M}_3'}{\text{M}'}$, three equations of condition would be, in strictness, required for the determination of the three unknown quantities ;—namely, the coefficient of the inverse cube of the distance, and the two parts of the coefficient of the inverse fifth power. However, the distance being greater than four times the length of the magnet, the angle of deflection, u, is always small, and the term involving the square of its sine may be neglected in comparison with the others. Accordingly, if we make, for abridgment,

$$\frac{2\text{M}}{\text{X}} = \text{Q}, \quad 2 \frac{\text{M}_3}{\text{M}} - 3 \frac{\text{M}_3'}{\text{M}'} = h,$$

the expression for the tangent of the angle of deflection is reduced to the form

$$\tan u = \frac{\text{Q}}{\text{D}^3} \left(1 + \frac{h}{\text{D}^2} \right).$$

In the method of deflection employed by Professor LAMONT, the deflecting bar is always perpendicular to the suspended bar. In this case, therefore, $\psi = 90°$, and the equation of equilibrium is reduced to

$$\text{X} \sin u = \frac{2\text{M}}{\text{D}^3} \left\{ 1 + \left(2 \frac{\text{M}_3}{\text{M}} - 3 \frac{\text{M}_3'}{\text{M}'} \right) \frac{1}{\text{D}^2} \right\} ;$$

and the equation is the same as that to which it is reduced in the former case, the *sine* of the angle of deflection being substituted for the *tangent*. It appears from the result, that this method is to be preferred to the former, not only because the angle of deflection is greater, *cæt. par.*, but also because the variable part in the coefficient of the inverse fifth power of the distance is strictly evanescent.

It remains now to inquire in what manner the quantity h, which expresses the ratio of the two coefficients, may be known *à priori;* and whether that quantity may be made to vanish, by any simple relation between the acting magnets.

For this purpose we must know, at least approximately, the law of magnetic distribution, or the function of r by which m is represented. Almost the only knowledge which we possess on this subject is that derived from the researches of Coulomb. From these researches M. Biot has inferred, that the quantity of free magnetism, in each point of a bar magnetized by the method of double touch, may be represented by the formula

$$m = \mathrm{A}\,(\mu^{l-r} - \mu^{l+r})\,;$$

μ being a quantity independent of the length of the magnet, and A a function of μ and l. M. Biot has further shown, that when the length of the magnet is small, the relation between m and r is approximately expressed by the simple formula

$$m = m' \frac{r}{l}\,;$$

the curve of intensities becoming, in that case, very nearly a right line passing through the centre of the magnet.

Employing then this approximate formula, we have

$$\mathrm{M} = \int_{-l}^{+l} mr dr = \frac{m'}{l} \int_{-l}^{+l} r^2 dr = \tfrac{2}{3} m' l^2\,;$$

$$\mathrm{M}_3 = \int_{-l}^{+} mr^3 dr = \frac{m'}{l} \int_{-l}^{+l} r^4 dr = \tfrac{2}{5} m' l^4.$$

The ratio of these quantities is independent of m', and we have simply

$$\frac{\mathrm{M}_3}{\mathrm{M}} = \tfrac{3}{5}\, l^2.$$

Finally, substituting in the value of h above given, and designating the half lengths of the deflecting and of the suspended magnets by l and l', respectively,

$$h = \tfrac{3}{5}\,(2l^2 - 3l'^2)\,;$$

a quantity whose value may be exactly known, independently of experiment. This quantity vanishes, when $l^2 = \frac{3}{2}\,l'^2$, or

$$l = 1 \cdot 224\, l';$$

a result which is independent of the magnetic state of the bars.*

As the preceding results depend, in part, upon an empirical law of magnetic distribution, which is only approximately true in the case of small magnets, it seemed desirable to obtain a confirmation of their accuracy by direct experiment. The nature of such confirmation will be immediately understood from the form of the relation between the angle of deflection and the distance. For since, in the method of deflection employed by GAUSS, $\mathrm{D}^3 \tan u = \mathrm{Q}\,(1 + h\mathrm{D}^{-2})$, the function $\mathrm{D}^3 \tan u$ will be constant for all values of D, when $h = 0$; while, if the coefficient of the fifth power of the distance have a sensible value, it will vary with D, its values forming a decreasing or increasing series, as D increases, according as h is positive or negative. Hence we have only to observe the deflections produced at different distances, when the two magnets have the relative lengths pointed out above, and to compare the results with those obtained under other circumstances.

Several series of experiments were accordingly made in the beginning of the present month, in some of which the lengths of the two magnets were the same, while in others they were in the deduced ratio of the number 1·224 to 1. The form of the magnets was cylindrical, their lengths being 3 inches, and

* If the centre of the deflecting magnet be in the magnetic meridian passing through the centre of the suspended magnet, and its axis perpendicular to the same line, we find, by a process similar to that above given, that the condition to be fulfilled, in order that the term involving the fifth power of the distance may vanish, is

$$\frac{\mathrm{M}_3}{\mathrm{M}} = 4\frac{\mathrm{M}_3'}{\mathrm{M}'}\,;$$

and accordingly that the corresponding relation between the lengths of the two magnets is, in that case,

$$l = 2l'.$$

$3\frac{2}{3}$ inches, and their diameter $\frac{3}{10}$ of inch. The suspended bar was hung by two fibres of untwisted silk, and inclosed in a small wooden box with glazed front. The deflections were observed by means of a mirror attached below the magnet, which reflected the divisions of a scale placed at a distance of nearly six feet from it. As the utmost precaution was required in the experiments, the use of a copper ring or metallic box was dispensed with, and the arc of vibration reduced by means of a magnet, which was always replaced carefully in the same position after use. The deflecting magnet was placed on the east and west sides of the suspended one, at distances varying from 15 to 30 inches. The distances were observed by the help of a standard scale, a line at the middle of the magnet being made to coincide with the image of the division of the scale, reflected from its polished side. The observations were made beginning with the longest, and proceeding to the shortest distance at one side, and back again in the reverse order at the other, so that the two observations at the same distance were taken at times equally remote from the middle epoch.

The angles of deflection were calculated by the formula

$$\tan 2u = \tfrac{1}{2}(n_e - n_w)\,k\,;$$

where n_e and n_w denote the observed readings of the scale, with the marked end of the deflecting bar to the east and to the west respectively. The value of the constant k is given by the formula

$$k = \frac{a}{d}\left(1 + \frac{\mathrm{H}}{\mathrm{F}}\right);$$

a denoting the length of one division of the scale, d its distance from the mirror, and $\frac{\mathrm{H}}{\mathrm{F}}$ the ratio of the torsion force to the magnetic force. In the present instance, $a = {\cdot}038935$ of inch; $d = 68{\cdot}52$ inches; and $\frac{\mathrm{H}}{\mathrm{F}} = {\cdot}000345$. Hence $\log k = 6{\cdot}75467$; and the angle corresponding to one division of the scale was consequently

$$\tfrac{1}{2}\tan^{-1} k = 58''{\cdot}623.$$

The following Tables exhibit the results of the observations. The first column of each contains the distances of the magnets, in feet; the second, the values of $\frac{1}{2}(n_e - n_w)$, these values being the means of those obtained with the deflecting magnet on the east and on the west side of the suspended magnet;

the third column contains the calculated values of u; and the fourth those of $D^3 \tan u$. In the observations of Series I., II., and III., the lengths of the deflecting and suspended magnets were in the ratio of the numbers 1·224 to 1; in those of Series IV. and V., the lengths of the two magnets were equal.

Series I. Time $11^h\ 35^m$ A.M. . . . $0^h\ 36^m$ P.M.
Magnet away, Scale reading = 496·5 . . . 494·5.

D	$\frac{1}{2}(n_e - n_w)$	u	$D^3 \tan u$
1·5	351·05	5° 38′ 32″·5	·33344
2·0	146·92	2° 23′ 13″	·33347
2·5	75·12	1° 13′ 21″	·33343

Series II. Time $11^h\ 37^m$ A.M. . . . $0^h\ 30^m$ P.M.
Magnet away, Scale reading = 495·9 . . . 495·0.

D	$\frac{1}{2}(n_e - n_w)$	u	$D^3 \tan u$
1·5	350·61	5° 38′ 7″·5	·33302
2·0	146·71	2° 23′ 1″	·33300
2·5	75·06	1° 13′ 17″·5	·33317

Series III. Time $11^h\ 5^m$ A.M. . . . $0^h\ 25^m$ P.M.
Magnet away, Scale reading = 496·4 . . . 494·3.

D	$\frac{1}{2}(n_e - n_w)$	u	$D^3 \tan u$
1·4167	417·24	6° 40′ 16″	·33254
1·8750	177·89	2° 53′ 13″	·33242
2·5000	74·96	1° 13′ 12″	·33275

Series IV. Time $1^h\ 30^m$ P.M. . . . $2^h\ 23^m$ P.M.
Magnet away, Scale reading = 494·4 . . . 494·6.

D	$\frac{1}{2}(n_e - n_w)$	u	$D^3 \tan u$
1·25	389·70	6° 14′ 42″·5	·21373
1·50	223·95	3° 37′ 38″	·21393
2·00	94·38	1° 32′ 7″·5	·21444
2·50	48·45	0° 47′ 19″·5	·21510

Series V. Time $1^h\,55^m$ P. M. . . . $3^h\,16^m$ P. M.
Magnet away, Scale reading $= 495 \cdot 7 \ldots 498 \cdot 3$.

D	$\frac{1}{2}(n_e - n_w)$	u	$D^3 \tan u$
1·25	368·46	5° 54′ 52″·5	·20233
1·50	211·99	3° 26′ 8″	·20261
1·75	133·41	2° 10′ 6″	·20292
2·00	89·39	1° 27′ 16″	·20313
2·25	63·00	1° 1′ 32″	·20390
2·50	45·99	0° 44′ 55″·5	·20420

These results verify the conclusions to which we have arrived above. The values of the function $D^3 \tan u$ are constant for all distances in the first three series, the differences in the resulting values being less than the probable errors of observation; and consequently, the coefficient of the inverse fifth power of the distance is insensible. In the fourth and fifth series, on the other hand, in which the lengths of the magnets are equal, the values of this function form an increasing series, as D increases; and therefore, in this case, the coefficient of the inverse fifth power of the distance has a sensible negative value.

We may further employ these results to test the accuracy of our conclusions, by deducing from them the values of the two coefficients, in the expression for the tangent of the angle of deflection, and comparing their ratio with that furnished by theory. It is needless to make this computation for the numbers of the first three series; for it is manifest from the results, that the second coefficient is insensible, as it theoretically should be. From the results of Series IV. we deduce by the method of least squares,

$$Q = \cdot 2148, \quad hQ = - \cdot 0017, \quad h = - \cdot 0078.$$

We obtain, in like manner, from the results of Series V.,

$$Q = \cdot 2037, \quad hQ = - \cdot 0022, \quad h = - \cdot 0110.$$

And the mean of the resulting values is $- \cdot 0094$. Now, in these two series, the length of each of the magnets was three inches; that is, $l = l' = \cdot 125$, the half lengths being expressed in feet. Substituting these values in the expression for h, it becomes

$$h = - \cdot 0094,$$

exactly agreeing with the mean of the experimental values.

We are justified in concluding therefore, that, in the case of small magnets, the ratio of the two coefficients may be inferred *à priori*, by the formula

$$h = \tfrac{3}{8}(2l^2 - 3l'^2);$$

and, consequently, that the coefficient Q, or $2\frac{M}{X}$, may be obtained, from the result of observation at a single distance, by the formula

$$Q = \frac{D^3 \tan u}{1 + hD^{-2}}.$$

It follows from this, as we have seen, that when $l = 1{\cdot}224\,l'$, $h = 0$, and the value of Q is reduced to

$$Q = D^3 \tan u.$$

POSTSCRIPT.—While the preceding pages were passing through the press, I received a memoir from Professor Lamont, on the determination of the earth's magnetic force in absolute measure, in which the author has proposed various modifications in the existing method, and has considered, with great minuteness of detail, the many corrections which are required in the immediate results of observation. Some of the conclusions of the present paper are, I find, thus anticipated; in particular, the form of the equation of equilibrium of the suspended magnet, for the case in which the axes of the two magnets are at right angles. Professor Lamont seems to have considered, however, that no approximation to the law of magnetic distribution was possible; and he has accordingly not thought of deducing *à priori* the ratio of the coefficients of the terms in the above-mentioned equation (except in the imaginary case in which the whole force is supposed to emanate from the two ends of the bars), or, therefore, of employing that ratio, as is proposed in the present paper, to supersede experiment, and thus evade the errors resulting from the process of elimination.

I take this opportunity of stating, that the present paper was, in substance, written during the last summer; and that several instruments have been constructed on the principle suggested in it. The delay in laying it before the Academy has arisen from the desire of obtaining, previously, an experimental confirmation of the accuracy of the results.

H. LL.